

Internal Exam 2024  
 P. K. R. M. College Dhanbani  
 Generic Math  
 Sem - 2

Q.1 Answer any three

- (1) State and prove Leibnitz's theorem  
 (2) using  $\epsilon$ - $\delta$  method, prove that the f<sup>n</sup>

$$f(x) = x^2 \sin \frac{1}{x} \quad x \neq 0$$

$$= 0 \quad x = 0$$

- (3) ~~If  $y = \frac{1+x^2}{\tan x}$~~  (3) If  $y = (\tan^{-1} x)^2$

Prove that  $(1+x^2)y'' + 2xy'(1+x^2)$

$$= 2$$

- (4) Evaluate the integral

$$\int_0^{\pi/2} \sin^n x \, dx$$

- (5) If  $\vec{v}(t)$  is a vector f<sup>n</sup> of constant variable  $t$  of constant magnitude

Prove that

(a)  $\vec{v} \cdot \frac{d\vec{v}}{dt} = 0$

(b)  $\vec{v} \times \frac{d\vec{v}}{dt} = 0$

Sem. II answer any three

Solve

(1)

$$\frac{dy}{dx} = e^{x+y} + x^2 e^x$$

(2)

$$\frac{dy}{dx} = \frac{x^2 + y^2 - 2xy}{2x^2}$$

(3)

$$p^2 - 7p + 12 = 0$$

where  $p = \frac{dy}{dx}$

(4)

Solve by Charpit method

(1)  $(p^2 + q^2)z = qz$



Sem - II

Answer any three

- ① Define Countable & uncountable set
- ② state and prove Bolzano-Weierstrass theorem
- ③ state prove Cauchy Root test
- ④ Test the convergence or divergence

$$U_n = \sqrt[n^4+1]} - \sqrt[n^4-1]}$$

- ⑤ Prove that

$$\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots + \frac{1}{n^p} + \dots$$

is convergent  $p > 1$  and div.  $p \leq 1$

Sem - IV Answer any three

- ① prove that every subgroup of a cyclic group is cyclic
- ② state and prove Lagrange's theorem
- ③ Define a ring with example
- ④ prove that every finite integral domain is a field
- ⑤ Define Normal subgroup